

EFFECT OF WALL CORRUGATIONS ON THE
LOWER HYBRID WAVE SPECTRUM OF A WAVEGUIDE ARRAY

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ABSTRACT

The effect of wall corrugations on the lower hybrid wave spectrum of a waveguide array is investigated in a linear approach for the conditions of the Alcator A experiment. It was found that a limited number of grooves placed on both sides of a two-waveguide grill can considerably alter the wave power spectrum launched into the plasma. However, if the set of the grooves is situated further than one wavelength from the grill its influence on the wave power spectrum is weak.

It is well known that RF plasma heating in the lower hybrid frequency range requires the excitation of slow waves, which can only penetrate into the plasma interior for large enough k_{\parallel} . A system consisting of an array of phased waveguides or grill was proposed by P. Lallia [1] to satisfy this criterion and has proved successful. M. Brambilla developed the theory of the grill and calculated its properties [2]. The results of this calculation were confirmed in a number of experiments [3]. However, in some cases the real experimental conditions differ considerably from those assumed theoretically in that the walls of the plasma chamber are often made of bellows. It was shown in [4] that the corrugation of the wall can influence the spectrum of excited waves if the number of radiating waveguides does not exceed two.

This work was undertaken to investigate the effect of corrugations on the Alcator A lower hybrid heating experiment. In this instance a two-waveguide grill is used for exciting lower hybrid waves at a frequency of 2.45 GHz. The simplified geometry of the experiment which was used for the calculation is shown in Fig. 1. The width of waveguide (b) is 1.275 cm and the wall between them (b_1) is 0.09 cm thick. The depth of the wall corrugation h is 1 cm and its period ($b_3 + b_4$) is 0.8 cm. It should be noted that the corrugation does not begin at the edge of waveguides but after a 12.5 cm smooth wall (b_2). The corrugation was incorporated into this calculation as a set of grooves with rectangular cross-section. The number of grooves (N_g) could be increased up to 10 at either side. All geometrical parameters were normalized to $\lambda = c/\omega$. The plasma density

dependence on x was assumed to be linear as $n/n_c = \delta(x - x_p)$ where $\omega = \omega_{pe}$ when $n = n_c$. The dimensionless parameter δ indicates the steepness of the density gradient and is chosen to be 1200. This value of δ corresponds to $\nabla n = 4.5 \times 10^{13} \text{ cm}^{-4}$, as measured in [5]. The plasma impedance, determined as $\rho = -E_z/B_y$, was employed to meet the boundary condition at the plasma surface. The analytical expression for it was obtained in [6]; when the accessibility criterion is satisfied and for $\delta \gg 1$ ($N_z = k_{\parallel} c/\omega$)

$$\rho \approx 1.2(1 + i/\sqrt{3}) (N_z^2 - 1)^{2/3} \delta^{-1/3}. \quad (1)$$

The effect of transformation of under-slowed waves into whistlers was neglected. Practically this means that no reflection from inner plasma layers is taken into account.

The method proposed by M. Brambilla [2] was used for calculation. The electromagnetic fields in the region $0 < x < x_p$ are written as a Fourier integral over the parallel refractive index N_z . The fields in the p^{th} waveguide are presented as a superposition of eigenmodes

$$\begin{aligned} E_z &= e^{i\Phi_p} \left\{ A_p e^{ix} + D_{op} e^{-ix} + \sum_{m=1}^{\infty} D_{mp} e^{\gamma_m x} \cos \left[\frac{m\pi}{b} (z - z_p) \right] \right\} \\ B_y &= e^{i\Phi_p} \left\{ -A_p e^{ix} + D_{op} e^{-ix} - i \sum_{m=1}^{\infty} \frac{D_{mp}}{\gamma_m} e^{\gamma_m x} \cos \left[\frac{m\pi}{b} (z - z_p) \right] \right\} \end{aligned} \quad (2)$$

where $\gamma_m = \sqrt{\left(\frac{m\pi}{b}\right)^2 - 1}$, A_p is the amplitude of incident wave, D_{op} of the reflected fundamental mode, D_{mp} of the reflected evanescent modes, and z_p is the position of the p^{th} waveguide. The fields in the grooves can be written as

$$\begin{aligned} E_z &= e^{i\phi_p} \left\{ D_{op} \sin(x+h) - i \sum_{m=1}^{\infty} D_{mp} \sinh \left[\gamma_m(x+h) \right] \cos \left[\frac{m\pi}{b} (z-z_p) \right] \right\} \\ B_y &= e^{i\phi_p} \left\{ i D_{op} \cos(x+h) - \sum_{m=1}^{\infty} \frac{D_{mp}}{\gamma_m} \cosh \left[\gamma_m(x+h) \right] \cos \left[\frac{m\pi}{b} (z-z_p) \right] \right\} \end{aligned} \quad (3)$$

To generalize the approach it is assumed that the chamber wall is characterized by a reactive impedance $Z_w = -i\beta$, which is defined as $Z_w = E_z/B_y|_{x=0}$. From the equations written above it is clear that for a ribbed wall, $Z_w = -i \tan(h)$ if the distance between ribs is small and if one can neglect the higher modes. The further calculations showed that this assumption was justified in most cases. Unlike the case with the smooth wall considered in [2], it now becomes impossible to use Fourier analysis for the expression for E_z in the $x = 0$ plane. Instead, one must use the function $F = E_z - Z_w B_y$, which is equal to zero in the $x = 0$ plane outside the throats of waveguides and grooves [4]. The rest of the procedure remains the same.

Imposing the continuity of electromagnetic fields at the throats of waveguides and grooves leads to a set of linear equations for the unknown amplitudes D_{mp} which is solved numerically. The capacity of the computer allowed us to take into account as many as 10 modes in each of a total of 30 grooves and waveguides; thus the maximum number of equations was 300. Having found these amplitudes it is possible to calculate the energy flux into the plasma for each Fourier component and obtain the power spectrum incident on the plasma [2-4,7].

The effect of wall corrugations in shaping the wave power spectrum is illustrated by considering a single narrow waveguide ($b = 0.3$) which is usually thought of as a source of a broad spectrum. Figure 2 shows the dependence of its power spectrum on the wall inductance β . It is seen

that the maximum in the spectrum shifts to greater N_z with growth of β . There is no singularity in these spectra since penetration of the waves into plasma weakens their amplitudes and plays the role of an effective dissipation process [4].

The narrow waveguide happens to be well matched with the narrow vacuum gap x_p , and therefore the level of reflectivity remains low. As seen from Fig. 3, when the gap gets wider the spectrum becomes sharper and its maximum shifts to smaller N_z ; at the same time the reflectivity also increases. These results are consistent with the data from [8], where the eigenmodes for the plasma-ribbed wall system were calculated.

In Fig. 4 the spectra of the two-waveguide grill used in Alcator A are graphed. In this case the two waveguides are driven out of phase ($\Delta\phi = \pi$). The spectra are calculated for two assumptions: (1) there are no grooves and the wall is smooth ($\beta = 0.001$) and (2) the wall is uniformly inductive ($\beta = 0.3$), with approximate correspondence to the real corrugation. The first spectrum reveals the singularity near $N_z = 1$, which is the evidence of excitation of unslowed surface waves. In the second case the maximum is located in the favorable region.

Figure 5 shows how the spectrum of the first type deforms when grooves appear on both sides of the grill. The rest of the wall is smooth ($\beta = 0.001$). It is observed that if the number of grooves is high enough (curve 5, $N_g = 10$) the spectrum practically coincides with the second type from Fig. 4.

Figure 6 shows the effect of increasing the distance b_2 between the waveguides and corrugations. When it becomes comparable to the vacuum wave length (curve 2, $b_2 = 5$), the influence of the corrugation on the

excitation of slow waves is weak. Unfortunately, the first (and very prominent) maximum occurs in the least accurate region of the under-slowed waves ($1 < N_z < 1.5$) due to our convention of not specifying the boundary condition for such waves.

The curves shown in Fig. 6 were verified in another approach. All of the wall except the smooth section b_2 was ascribed to by the inductance $\beta = 0.3$. The spectra obtained in this way for both in-phase and out-of-phase excitation are shown in Fig. 7. Curve 1 (out-of-phase excitation) is very similar to the second case of Fig. 6. Curve 2 illustrates the strong decrease in wave power with $N_z > 2$ for in-phase excitation.

In conclusion, the effect of wall corrugations on the power spectrum of a two waveguide array has been calculated. The corrugations can markedly alter the power spectrum when they are positioned close to the waveguide mouth. However, when they are more than one free space wavelength away their effect is markedly reduced.

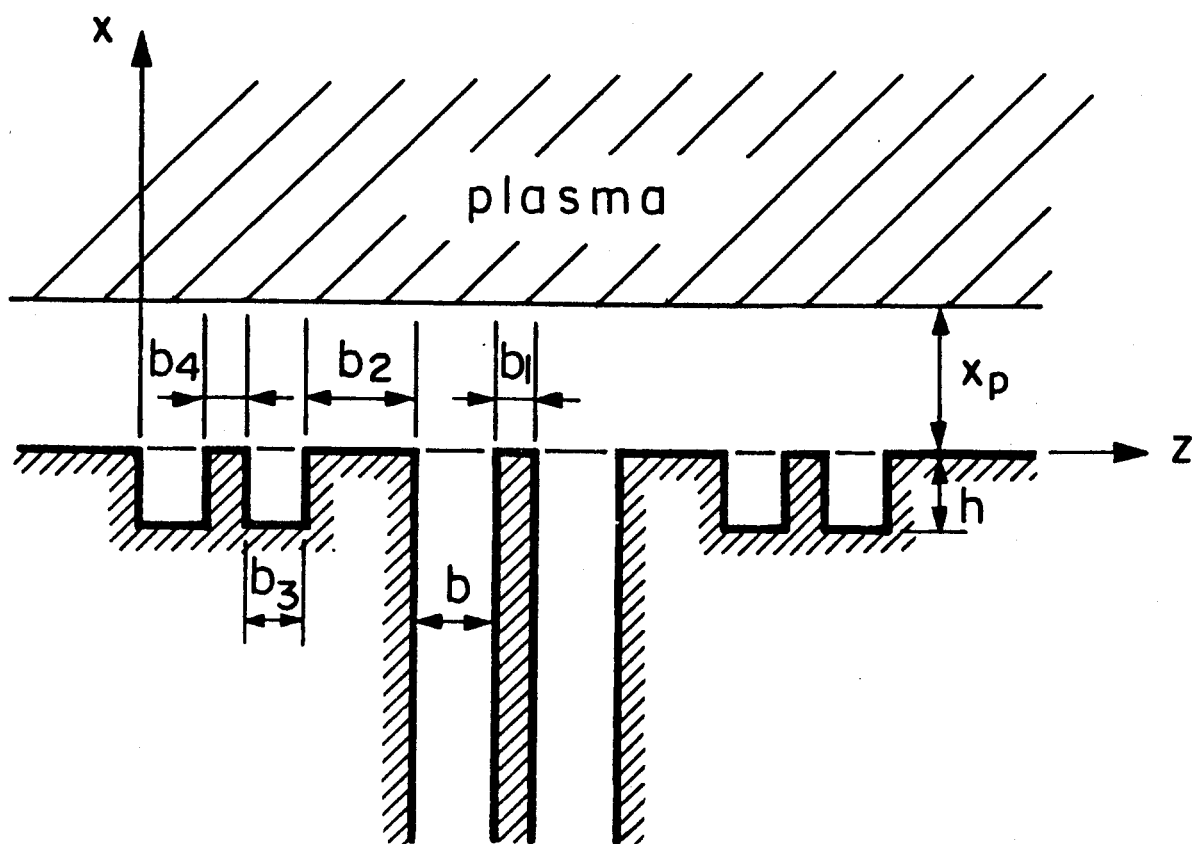
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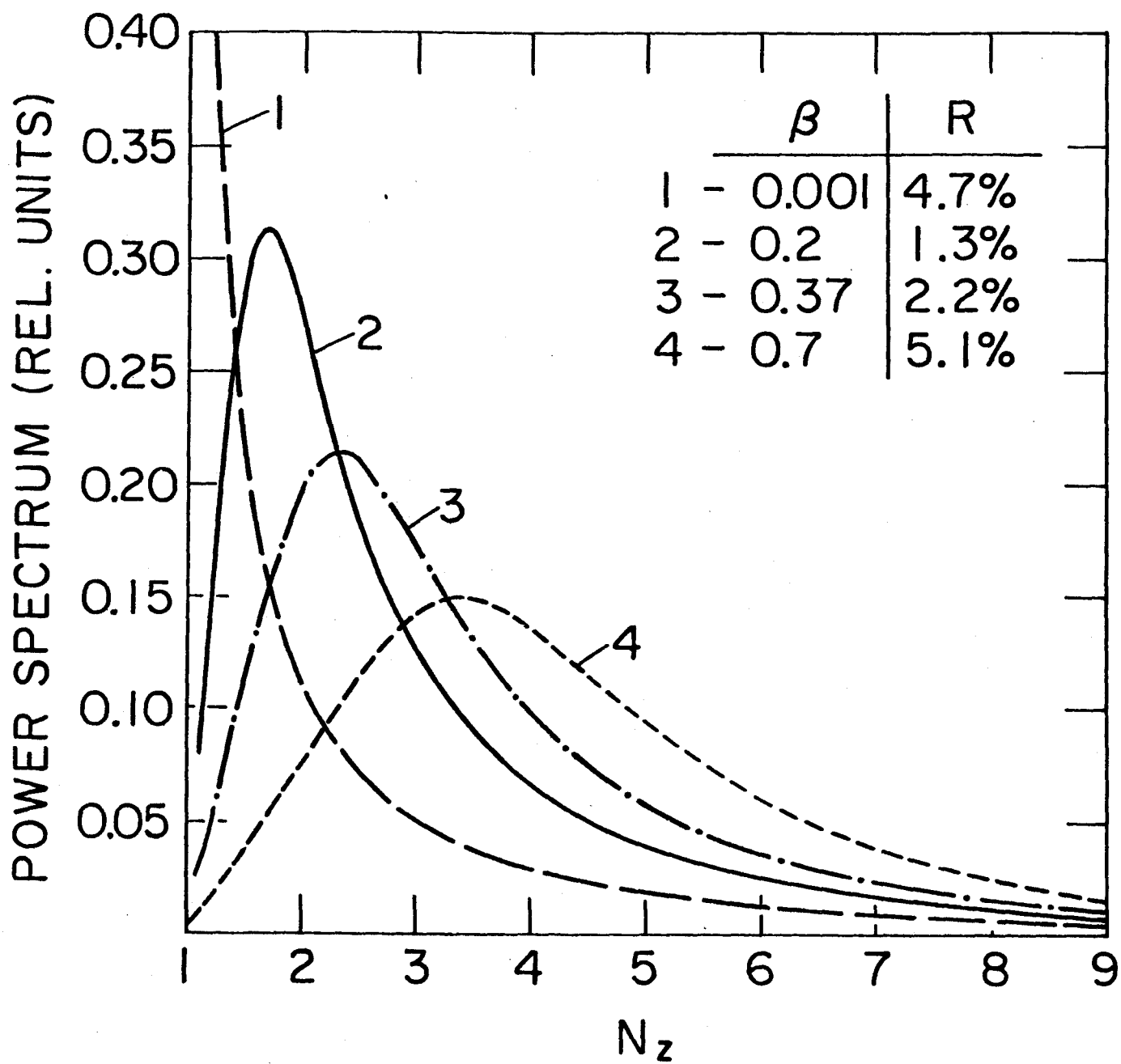
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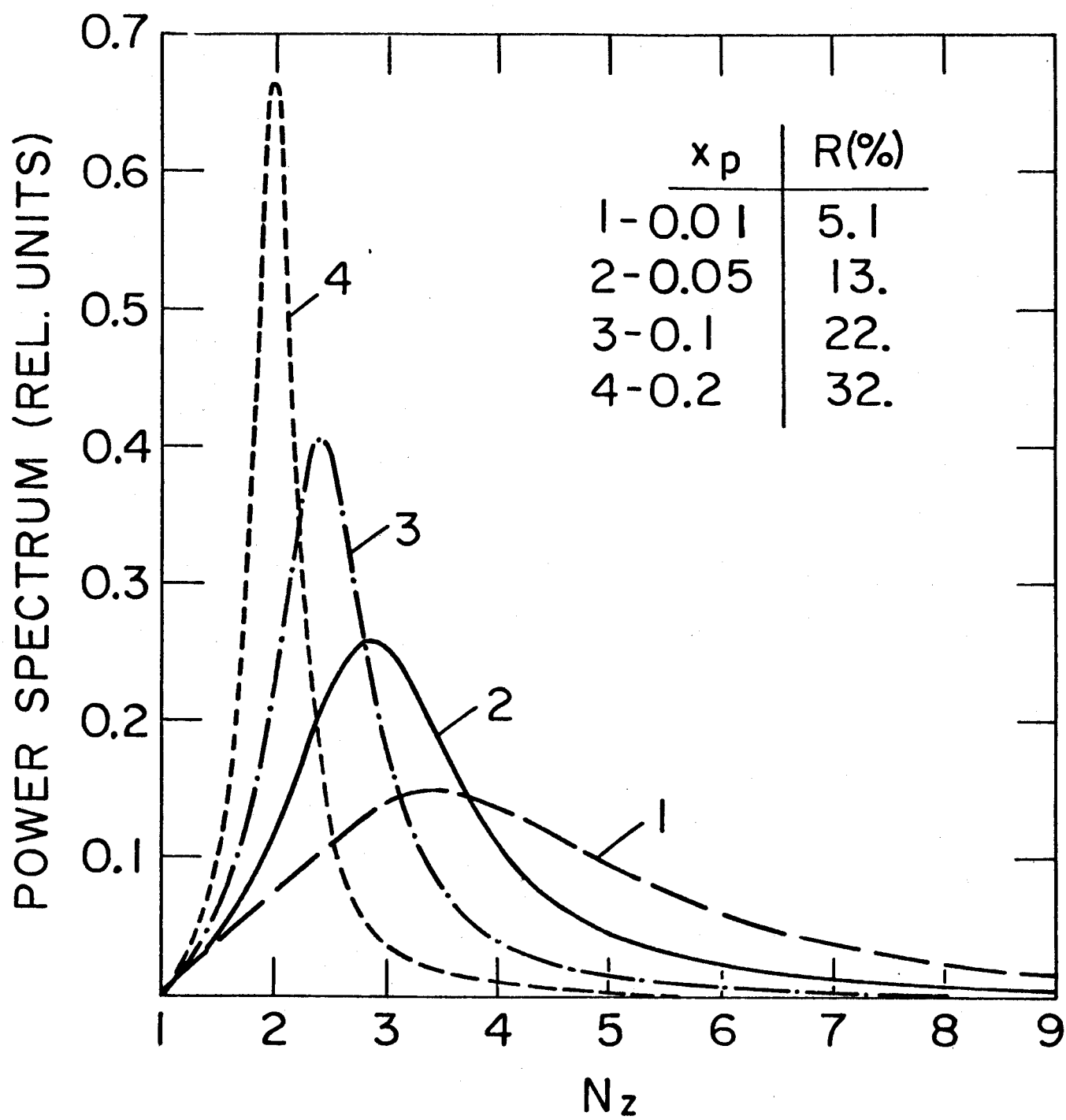
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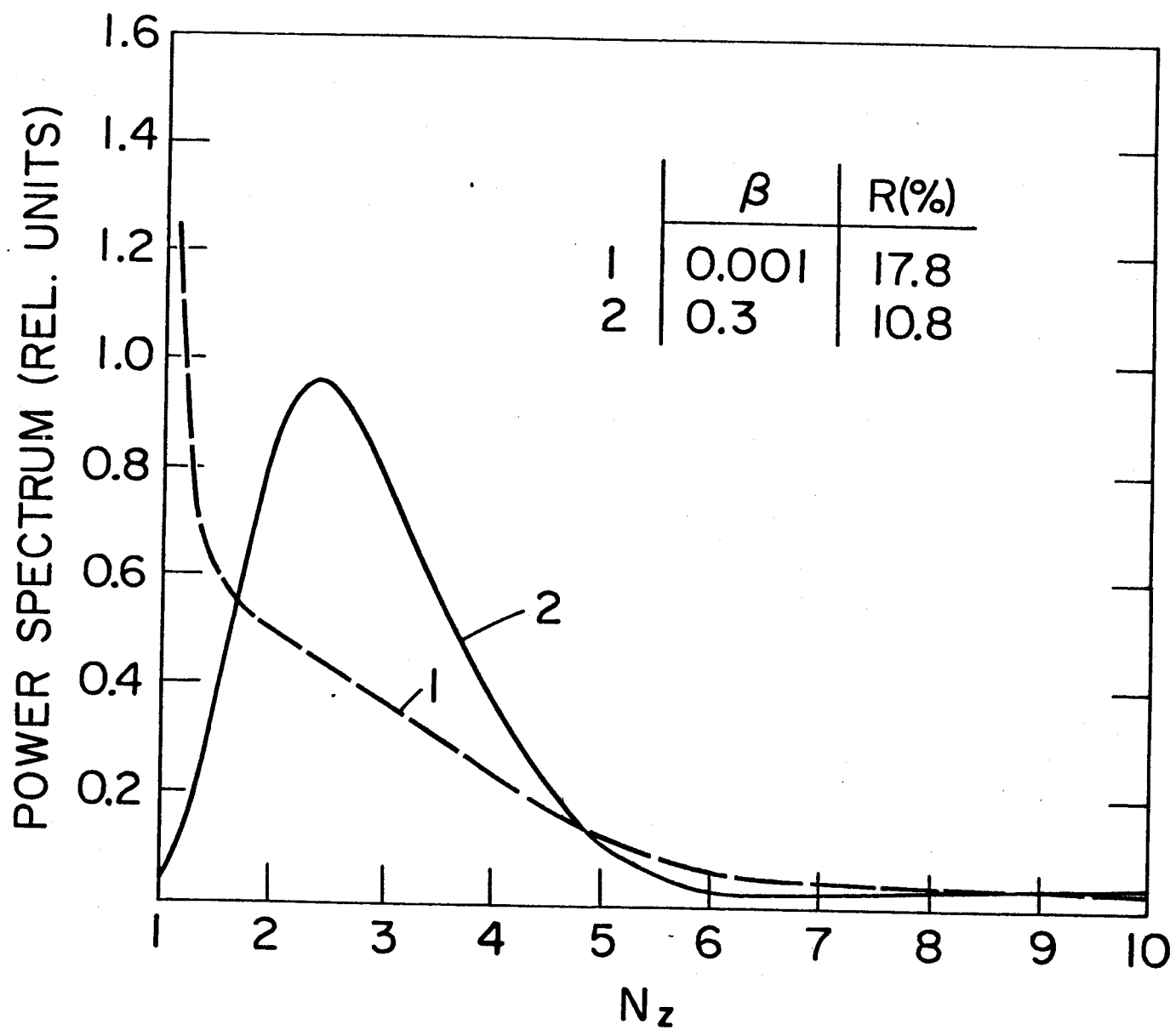
FIGURE CAPTIONS

- Figure 1. Schematic presentation of the experimental geometry taken for the calculation.
- Figure 2. Dependence of the power spectrum excited by a single waveguide ($b = 0.3$) on the wall inductance; $x_p = 0.01$:
1 - $\beta = 0.001$; 2 - $\beta = 0.2$; 3 - $\beta = 0.37$; 4 - $\beta = 0.7$.
 R is the total reflected power of the waveguide.
- Figure 3. Dependence of the power spectra excited by the waveguide of Fig. 2 on x_p . $\beta = 0.7$; 1 - $x_p = 0.01$; 2 - $x_p = 0.05$;
3 - $x_p = 0.1$; 4 - $x_p = 0.2$.
- Figure 4. The power spectra excited by a two-waveguide grill
 $b = 0.654$; $b_1 = 0.046$; $x_p = 0.01$ and $\Delta\phi = \pi$.
1 - $\beta = 0.01$ and 2 - $\beta = 0.3$
- Figure 5. The power spectra excited by the same grill with the grooves on both sides. $b_2 = 0.2$; $b_3 = 0.2$; $b_4 = 0.2$; $\beta = 0.001$;
1 - $N_g = 1$; 2 - $N_g = 2$; 3 - $N_g = 4$; 4 - $N_g = 10$.
- Figure 6. Dependence of the power spectra excited by the grill of Fig. 5 on the smooth wall length. There are 10 grooves on both sides. 1 - $b_2 = 1.0$ and 2 - $b_2 = 5.0$. In both cases $b_3 = 0.2 = b_4$ and $\beta = 0.001$.
- Figure 7. The power spectrum in case of the uniformly inductive wall ($\beta = 0.3$) except for a smooth section $b_2 = 5$.
1 - $\Delta\phi = \pi$; 2 - $\Delta\phi = 0$.









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